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Hyperbolic Discounting in Economic Modelling

Michal Andrle
Jan Brůha

INSTITUT PRO EKONOMICKOU A EKOLOGICKOU POLITIKU

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VYSOKÁ ŠKOLA EKONOMICKÁ V PRAZE – FAKULTA NÁRODOHOSPODÁŘSKÁ

Institut pro ekonomickou a ekologickou politiku
Vysoká škola ekonomická v Praze – Fakulta národohospodářská
Katedra hospodářské politiky

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Hyperbolic Discounting in Economic Modelling

Michal Andrle [*]
Jan Brůha[**]

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[*] University of Economics, Prague – andrle@sorry.vse.cz
[**] CERGE-EI, jan.bruha@cerge-ei.cz

Abstrakt

The paper presents microeconomic foundations of intertemporal utility function and surveys exponential discounting with its implications. We explore the concept of hyperbolic discounting on the basis of normative and positive economics. The main implication of hyperbolic discounting is time inconsistent behavior of economic agents. This fact may have interesting implications for the economic policy. We discuss the issues of savings and consumption and environmental economics.

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Content

"…there may be a strong difference between an enjoyment which offers itself at the very moment and one which does not; while, on the other hand, there may be a very small difference, or no difference at all, between an enjoyment which is pretty far away, and one which is further away."
Böhm-Bawerk (1959, pp. 257)

I. Introduction

The neoclassical mainstream is based on several methodological keystones: one of the most important suggests that the behavior of economic agents can be described as if these agents solve an optimization problem. It is assumed that households behave as if they maximize their utility subject to a set of budget constraints, the utility being assumed to be a function of private consumption and possibly (depending on an application) of other arguments as well (public goods, leisure etc.) The firms are assumed to behave as if they maximize their objective (mostly profit) subject to technological constraints. This methodological approach enables testable restrictions to be derived.

For economists it is important to investigate the behavior of an economic system in time. Thus, it is necessary to extend the static analysis of household/firm behavior to situations, in which the underlying maximization problem is not static, but an intertemporal one. While the main issue in static problems is allocations of an income among expenditures on *n* types of goods, intertemporal situations usually involve questions of allocations of resources available during a time period to instants of that time period.

Thus it is necessary to make assumptions about evaluation of objectives (instantaneous utility, period profits) in distinct time instants by economic agents. Interest rates are a common tool for time evaluation of cash flows in market economies. However for utility evaluation there is not available any similar ‚objective' tool, since the consumer / household utility is at least not directly observable if it at all means something more than a useful analytical tool to household behavior modeling.

Mainstream economists (since Kenneth Arrow and Gerard Debreu have explicitly introduced intertemporal aspects into the general equilibrium paradigm) model consumption of (objectively identical) goods in different time instants as consumption of different goods. This implies that modeling of intertemporal household behavior requires assumptions about households' valuation of their utility in distinct time instants. A rule, which serves for this purpose, is called discounting. However, this rule should not be confused with cash flow discounting, since here the issue is a modeling of agents‘ preference structure.

A useful way of intertemporal utility function modeling is to define it as a time-dependent function of instantaneous utilities. Nowadays, a predominant approach is to define the intertemporal utility function as a discounted sum of instantaneous utilities, most often using an exponential discounting. This approach involves many interesting assumptions and implications, which will be partially addressed in this paper. It would be demonstrated that the exponential discounting is a very special case, and implies some intertemporal behavioral patterns, which seem to be at odds with empirical regularities both at the macro and micro level.

The intertemporal utility issues are not only important for positive economics (to find a satisfying way of economic agents behavior modeling), but for normative economics too. Normative implications of intertemporal valuations are discussed mostly in connection with environmental economics (valuation of ecological projects, natural resources exhaustion) and with fiscal policy (inter-generation altruism).

This paper is a summary paper on hyperbolic discounting for a novice reader. The rest of the paper is organized as follows: Section 2 reviews historical approaches to discounting. Section 3 defines the concept of hyperbolic discounting, while Section 4 discusses its importance for consumer theory. Section 5 is concerned with the hyperbolic discounting implications for normative assessment of environmental economics and the last section concludes.

II. Historical approaches

This section reviews historical approaches to discounting. While early economists (e.g. Böhm-Bawerk, A. C. Pigou or I. Fischer based on observed discounting claimed that people are myopic) occasionally discussed these issues, a real development had waited for pioneering contributions by Ramsey (1928) and Samuelson (1937).

It is possible to identify two main reasons, why discounting is an important part of the standard economics toolbox and why attention to an appropriate approach to discounting is discussed in many branches of contemporary economics, including macroeconomic modeling (RBC models, growth models) and environmental economics. Positive aspects involve a proper way of economic agents intertemporal modeling, while from the normative point of view, a fair approach to intertemporal valuation of welfare, costs or utilities in a social welfare function is the goal.

Indeed, a first application of discounting was a construction of intertemporal welfare in Ramsey's (1928) optimal growth model. Interestingly, Ramsey, from an ethical perspective, argued against discounting of the future welfare. Thus, his paper is a good example of normative dealing with intertemporal issues. An important question, he posed, whether to incorporate any kind of discounting into a social welfare function (SWF) still remains.

A natural pro- argument is that discounting expresses a preference relation of sovereign economic agents and thus avoiding discounting in a SWF overlooks the preferences of people in a society. But, this normative approach rises a problem: whose preferences should be considered? Preferences of a current generation or of future generations, who do not live yet and thus it is impossible to know their preferences. There is another important factor: people usually express different discount factors which they would like to use for individual and social (governmental) decisions[1] (Henderson, Langford 1998, Cropper et. al, 1992).

A completely different perspective is investigated in positive economics: the goal is to find a proper way of intertemporal behavior modeling. From the positive perspective, the discounting has implications for consumer behavior, life cycle theory etc. A slightly different argument, why the discounting has to be used in macroeconomic modeling is a nowadays-standard argument of inter-generation altruism within long-lived dynasties. Barro (1974) argues that the utility of a current generation depends on the utility of its offspring, which depends on the utility of its offspring etc. Thus households behave as if they were infinitely lived since they care about the utility of its children and of children of its ad infinitum[2]. Of course, it is crucial not just how strong the intergeneration altruism is, but also the functional form of the discounting function is important. The functional form need not be exponential as in pioneering Barro‘s contribution; e.g. Phelps and Pollak (1968) use in their model of an impure intergenerational altruism quasi-hyperbolic discount function.

It is crucial to distinguish among various applications of discounting, since interpretations of a particular discount function differ according to an issue: if it is concerned with behavior of a sovereign consumer, with a positive application of the intergeneration altruism in economic growth theory or with a policy evaluation using a SWF. The most important distinction is between a problem of positive and normative economics.

From a positive perspective, the usage of the discounting concept requires a rigorous microeconomic treatment, a task done by Koopmans (1960). The micro-foundation of the intertemporal utility function can be based on both cardinal and ordinal utility theory. Koopmans (1960) presents a cost of deriving an ordinary intertemporal function based on instantaneous utilities using a concept of impatience. The impatience concept had been before discussed by Böhm-Bawerk (1959) or Fischer (1930), however the notion can be found also in work of very early economists see inter alia Smith (1759), Rae (1834).

In next few paragraphs, the text follows Koopmans’ idea of intertemporal utility function derivation. Denote a commodity basket (henceforth called program) as an infinite vector:

$$ {}_1x = (x_1, x_2, \ldots, x_t) = (x_1, {}_2x), \qquad (1) $$

where x_t is a consumption bundle in time t, and the expression ${}_tx$ denotes a program from time t perspective. This program can be decomposed as in (1). The goal is to define a utility function, which can be defined over a set of all possible programs. Such a function would represents

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[1] This applies especially to long-term decisions (environmental policy).

[2] The infinite lifetime is a usual assumption used in neoclassical models, including growth theory, see Barro and Sala-i-Martin (1995), and the RBC research, see Cooley (1994). Laitner and Juster (1993) empirically justify this assumption.

preferences, i.e. $U({}_1x') \geq U({}_1x'')$ if a decision-maker weakly prefers a program 1x′ to a concurrent program 1x′′.

Let U be an intertemporal utility function, i.e. let U be a function of instantaneous utility functions $u_t(x_t)$. An instantaneous utility function u satisfies that if a decision-maker at time t (weakly) prefers a bundle xt′ to a bundle xt′′ then $u(x_t') \geq u(x_t'')$. Koopmans (1960) proves that under a few assumptions, it is possible to define an intertemporal utility function in the following form[3]

$$U({}_1x) = \sum_{t=1}^{\infty} \beta^{t-1} u(x_t). \qquad 0 < \beta < 1 \tag{2a}$$

However, the functional form (2a) is cardinal, since the preference ordering is not invariant with respect to non-linear transformations of the instantaneous utility function u. The criterion (2a) has a natural analogy in continuous time, a preference ordering of a consumption flow $\{x'(t)\}_{t\in[0,\infty)}$ is based on the following functional:

$$U(x') = \int_0^{\infty} e^{-\rho t} u(x'(t))dt, \quad \text{for } \rho \geq 0. \tag{2b}$$

While in discrete time (2a), the higher value of □ implies patience, in continuous time setting (2b), patience is implied by a low value of the parameter □. It is possible to heuristically demonstrate the relationship between these two parameters as follows: consider a consumption flow $\{x'(t)\}_{t\in[0,\infty)}$ satisfying $x'(s) = x_t \ \text{ for } s \in [t-1, t)$. After some simple algebra we have:

$$U(x') = \int_0^{\infty} e^{-\rho t} u(x'(t))dt = \sum_{t=1}^{\infty} u(x_t) \int_{t-1}^{t} e^{-\rho s} ds = \frac{1-e^{-\rho}}{\rho} \sum_{t=1}^{\infty} \left(e^{-\rho}\right)^{t-1} u(x_t),$$

i.e. a counterpart of the parameter □ is $-\log \beta$. However, have in mind that this is a heuristics only.

The model (2) of the exponential discounting was used earlier on. Samuelson (1937) suggested it as very simple and elegant, his main concern was mathematical tractability and realism was secondary[4]. Koopmans (1960) firstly identified a set of axioms from which it is possible to derive (2), but still he did not consider this form of discounting as the only possibility.

The functional form (2) is very mathematically elegant and this can at least partially explain why is so popular among economists. However Ramsey (1928) considered discounting as ethically indefensible, this view can be formally expressed as □=1 in (2a) or □=0 in (2b) in a SWF. This rises a technical problem since then the value (2) is not bounded. Indeed, consider the following example: apples are the only consumption goods and a consumer always prefers more apples to less. Let the consumer be patient, i.e. □ = 1. Without loss of any generality assume that an instantaneous function u satisfies u(2) > u(1) > 0. The only argument is the number of apples eaten during an instant of time. Consider two programs x1 = (1,1,1,1,1, …) and x2 = (2,1,1,1,1, …). It is natural to expect that the consumer would prefer the program x2 to the program x1. However U(x1) is not possible to compare to U(x2), since both values are unbounded.

Thus we need another criterion, which would induce preference relation on a set of all programs. The following criterion was proved as useful: a program x′′ is weakly preferred to program x′ if and only if:

$$\sum_{t=1}^{\infty} \beta^{t-1}\left[u\left(x_t^{'}\right)-u\left(x_t^{''}\right)\right] \leq 0$$, in discrete time,

$$\int_0^{\infty} e^{-\rho t}\left[u\left(x'(t)\right)-u\left(x''(t)\right)\right]dt \leq 0$$ in continuous time.

It is easy to show that these criteria are more general then the criteria based on (2) and that they can be used in even more complicated cases then the illustrative apple example is.

It is useful to address the fundamental properties of the intertemporal utility function by Koopmans (1960), of which a special case is (2). This enables us to define and illustrate concepts as impatience, time-consistency or recursive structure of preferences. Koopmans (1960) specifies

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[3] For simplicity of exposition the infinite horizon is assumed, but nothing crucial would change if the finite horizon were considered.

four main assumptions, from which he derived the intertemporal utility function: (i) existence and continuity, (ii) sensitivity, (iii) weak non-complementarity and (iv) stationarity.[5] See Koopmans (1960) for a detailed exposition, here these assumptions are briefly discussed. .

(i) Existence and continuity: there exists a continuous function $U({}_1x)$ defined for all admissible programs ${}_1x = (x_1, x_2, \ldots,)$, such that for all t, x_t is a point of a connected subset of X, n-dimensional space of commodities. Existence is a rather natural assumption, while continuity models the intuitive requirement that if two programs are similar then their valuation is closed.

(ii) Sensitivity: there are two (first period) consumption bundles $x_1, x_1^{'}$ and a program ${}_2x$, such that

$$U(x_1, {}_2x) > U(x_1^{'}, {}_2x). \tag{3}$$

This implies that the intertemporal utility function is not constant over a set of all admissible alternative programs.

(iii-a) Weak noncomlementarity: for all $x_1, x_1^{'}, {}_2x, {}_2x'$

$$U(x_1, {}_2x) \geq U(x_1^{'}, {}_2x) \text{ implies } U(x_1, {}_2x') \geq U(x_1^{'}, {}_2x'), \tag{4a}$$

$$U(x_1, {}_2x) \geq U(x_1, {}_2x') \text{ implies } U(x_1^{'}, {}_2x) \geq U(x_1^{'}, {}_2x'). \tag{4b}$$

This assumption is relatively strong. Koopmans (1960) and Samuelson (1959) argue that its realism can be challenged. Anyhow, the relationships (4) imply that the intertemporal utility function can be written as

$$U({}_1x) = V(u_1(x_1), U_2({}_2x)), \tag{5}$$

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[4] „It is completely arbitrary to assume that the individual behaves so as to maximize an integral of the form envisaged in [the model]" Samuelson (1937, pp. 159).

[5] He actually used 5 assumptions, but the last one (existence of extreme programs) serves only as an anchor for the utility scale.

where $V(.)$ is an increasing function of both arguments. The last step is the assumption of stationarity, which restricts the temporal valuation:

(iv) Stacionarity: For fixed x_1 and all ${}_2x, {}_2x'$

$$U(x_1, {}_2x) \geq U(x_1, {}_2x') \text{ only if } U({}_2x) \geq U({}_2x'). \tag{6}$$

An important consequence is that calendar time does not have any impact on preferences. Thus the time index of the function V can be eliminated and this function satisfies the functional relationship: $U({}_1x) = V(u(x_1), U({}_2x))$. From this, the recursive structure of the intertemporal utility function is obtained, since the function also satisfies

$$U({}_1x) = V(u(x_1), u(x_2), \ldots, u(x_\tau), U({}_{\tau+1}x)), \tag{7}$$

for any □.

Koopmans (1960) proved that only under the following assumption, it is possible to find a monotone transformation of the function $U({}_1x)$ in the form (2).

(iii-b) Time-independence: for all $x_1, x_2, {}_3x, x_1', x_2', {}_3x'$ it holds that

$$U(x_1, x_2, {}_3x) \geq U(x_1', x_2', {}_3x) \text{ implies } U(x_1, x_2, {}_3x') \geq U(x_1', x_2', {}_3x'), \tag{9a}$$

$$U(x_1, x_2, {}_3x) \geq U(x_1', x_2, {}_3x') \text{ implies } U(x_1, x_2', {}_3x) \geq U(x_1', x_2', {}_3x'). \tag{9b}$$

The assumption (9) dictates that the marginal rate of substitution between two next instants is independent on future periods.

A consumer determines an optimal feasible program at every instant of time and thus when a new relevant piece of information arrives, her optimal program changes. This is rather obvious and economic theory provides a variety of tools for modeling such situations. A less obvious fact is that the optimal programs can be different even if there is not any new piece of information. The reason is that the discount function changes in time and „tomorrow“ will be „today“ tomorrow. Thus economic agents can perceive the same program differently even without any shock to preferences or fundamentals. Such a problem was firstly analyzed by Strotz (1956) and then further by Pollak (1968) and Peleg and Yaari (1973).

A behavior is a sequence of optimal programs and thus it is completely rational. However such a sequence need not be time-consistent, i.e. an optimal program ex-ante need not be an optimal program ex-post. Formally, we can define time-consistent preferences as follows:

$$U_{t+1}(x_{t+1}, {}_{t+2}x) \geq U_{t+1}(x_{t+1}{}', {}_{t+2}x') \text{ if } x_t = x_t{}' \text{ and } U_t(x_t, {}_{t+1}x) \geq U_t(x_t{}', {}_{t+1}x') . \qquad (10)$$

An agent with time-inconsistent preferences has two main possibilities: (i) she can try to commit to future actions, using rules or binding commitments etc., or (ii) not to commit and to choose a plan, which would be consistent with her future preferences. Strotz (1956) proved that the exponential discounting is a time-consistent intertemporal preference relation, which is rather a special property among intertemporal utility functions.

The exponential discounting function started to be used as an arbitrary approach to intertemporal valuation and the main reason being mathematical tractability. Then it was proved that it expresses time consistent preferences and based on rigorous microeconomic theory it was derived from a set of rigorous axioms and assumptions. Nowadays exponential discounting is a dominant approach used by mainstream economists and many famous and widely used macroeconomic textbooks – Sargent (1987), Lucas and Stokey (1989), Sargent and Ljungquist (2000) – even do not discuss alternatives to (2). Also, it is rather rare to find an explicit discussion what form of discounting to choose in a particular type of problem.

While the functional form (2) is dominant, its certain implications seem to be at odds, with observed empirical regularities. These ‚empirical‘ failures were investigated especially in two fields: (i) consumer theory and (ii) macroeconomics. These failures are discussed in next chapters.

III. Hyperbolic Discounting

Exponential discounting has become a standard in modern economics. Since the 60's it has been also formalized and incorporated into consumer theory. Nowadays, there are growing debates about the properties and consequences of the exponential discounting contradicting empirical analyses. We focus on the alternative way of discounting, (quasi-) hyperbolic discounting, analyzed mostly by Laibson (1996, 1997,a, 1997b) in the theory of consumption functions and Phelps and Pollack (1968) in the intergenerational altruism problem.

Again, we have to distinguish which stream of reasoning in favor of discounting we are considering, i.e. whether we analyze a social planner, intergenerational altruism or representative agent problem. Not surprisingly, the use of the discounting has a different interpretation depending on the type of the problem. In this part of the paper we consider mainly the individual consumer problem.

Although the exponential discounting has been embedded into the consumer theory, the functional form itself originated more or less on an ad-hoc basis, later formalized by Koopmans. Recent empirical research – i.e. Ainslie (1975, 1991), Loewenstein and Thaler (1989), Loewenstein and Prelec (1992) and other – reveals many interesting aspects of consumers' intertemporal decisions which are still usually absent in economic models. One of the problems[6] is the possibility of hyperbolic discounting and time inconsistency in intertemporal decisions of economic agents, mentioned by Strotz (1956). However, such an observation of time-inconsistent behavior is not new as can be seen from the quotation by Bohm-Bawerk at the very beginning of the paper.

It seems that economic agents, being impatient, do not discount using a constant rate. There are reasons to believe that individual preference for smaller reward to larger, but more distant in time, is different just because of the passage of time, although the time distance between both rewards remains constant. Empirical studies convincingly present that if there are two rewards sufficiently distant in time, agents display relative patience, i.e. they prefer more distant, but larger reward (two apples in 101 days) to smaller but earlier reward (one apple in 100 days).

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[6] Loewenstein and Prelec (1992) incorporate these „anomalies" of the standard model of the intertemporal choice into economic theory. Among possible explanations, they refer not only to the hyperbolic discounting, but they

However, as the day of the reward arrives, we can observe preference reversal in favor of higher impatience. Now, they prefer smaller, but earlier reward (one apple now) to more distant, but higher reward (two apples tomorrow). Such observations suggest that agents do not discount by constant rate, but by the rate that is decreasing function of time. In literature, this behavior is captured often by the hyperbolic function $(1+\alpha t)^{-\gamma/\alpha}$, e.g. Loewenstein and Prelec (1992). The discount rate is defined as the rate of decline of the discount function $f(t)$ in time; hence we may express the discount rate as

$$-\frac{\partial f(t)/\partial t}{f(t)}. \tag{11}$$

For the exponential discounting case (2a) the discount rate is constant $-\log\beta$. However for the case of hyperbolic discounting the discounting rate is a function of time: $\gamma/(1+\alpha t)$. In the short term the discount rate equals γ, in long term it converges to zero. This assumption corresponds to the above-mentioned observations. The hyperbolic function, however, lacks the mathematical elegance of exponential discounting. That is why economists use so called quasi-hyperbolic discount function with discount factors $\{1,\alpha\delta,\alpha\delta^2,\alpha\delta^3,\ldots\}$, which features qualitative properties of the hyperbolic function (i.e. a declining discount rate in time). Thus an alternative to (2a) is

$$U({}_1x)=u(x_t)+\alpha\sum_{i=1}^{\infty}\delta^i u(x_{t+i}),\ \text{where}\quad \alpha\in[0,1],\ \delta\in[0,1), \tag{12}$$

which for $\alpha=1$ reduces into the standard exponential discount function. The functional form of the quasi-hyperbolic[7] discounting was introduced by Phelps and Pollack (1968) in the model of intergenerational altruism. Laibson (1997) used the same functional form in the problem of intertemporal decision making of individual agent as a substitute for the hyperbolic function.

The expression (12) and other forms of the hyperbolic discounting imply time inconsistency in the intertemporal decisions. Due to passage of time the agent's view of optimal program changes. A consumer in time t values a future utility stream as follows:

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discuss also asymmetry in discounting gains and losses, reference point importance, loss aversion or the relevance of absolute magnitude of the benefit.

$$u(x_t) + \alpha\delta u(x_{t+1}) + \alpha\delta^2 u(x_{t+2}) + \dots, \tag{13a}$$

while the consumer in $t+1$ values the same stream as:

$$u(x_{t+1}) + \alpha\delta u(x_{t+2}) + \alpha\delta^2 u(x_{t+3}) + \dots. \tag{13b}$$

Hence in time t the ratio of marginal utilities of consumption of goods x_{t+1} and x_{t+2} equals to $\delta mu(x_{t+1})/ mu(x_{t+2})$, while one period later, in time $t+1$ the same ratio equals to $\alpha\delta mu(x_{t+1})/ mu(x_{t+2})$. These expressions differ in α, although no 'objective' condition changed. However, this change in marginal utility has consequences for the optimal allocation of an agent's resources. The optimal allocation of resources in $t+1$ is different than planned in t, even without occurrence of any new piece of information.

We can illustrate time inconsistency on well-known examples from everyday life – time inconsistency invokes possibility of regretting. In the long run, one declares the desire to be in good shape, prepare on exams and makes ambitious plans, however often one ends by overeating sweets, studying the last day before exam and all ambitious plans are given up. Time inconsistent preferences imply the problems of self-control and procrastination.

We shall distinguish two types of hyperbolic consumer: (i) naïve and (ii) sophisticated. The naïve hyperbolic consumer is not aware of her preferences, repeatedly re-optimizing her behavior. Hence, she repeatedly beliefs that although not today, but tomorrow, certainly tomorrow she starts her saving (training, etc.) program. In contrast, a sophisticated hyperbolic consumer is aware of her intertemporal conflict and tries to solve it.

A possible solution to the problem is a certain kind of commitment, either (i) internal or (ii) external. Already Strotz (1956) stressed the importance of the possibility of commitment. Recent literature (mainly on consumption and savings) focuses on mechanisms of external commitment. However Smith (1759) points out mechanisms of internal commitment and the power of will. The absence of a commitment technology may have serious consequences for consumer behavior. It

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[7] Note: We call discounting defined by (2) exponential and by (12) quasi-hyperbolic, although it is now hyperbolic functional form. The reason is that we try to express the qualitative properties of the function.

is the access to a commitment technology, which may have significant consequences for predictions of models of savings behavior or decentralized growth with renewable resources.

Laibson (1997a) and others treat the problem of the sophisticated hyperbolic consumer as an *intrapersonal* game among different 'selves' in time. Consumer's 'self' in *t* tries to commit future behaviour of *t+1* 'self' by adopting such contracts that are rather costly for the *t+1* 'self' to cancel. The behaviour of naïve and sophisticated hyperbolic consumers differs dramatically, since both types are extremes. Empirical observations suggest that even for a "potentially sophisticated" agent it takes time to get rid off a certain degree of naïveté.[8]

IV. Private consumption, savings and hyperbolic preferences

Empirical research – e.g. O'Donoghue and Rabin (1998, 1999)—shows for a significant part of households, that these households save less than they consider that they 'should' save for the post-productive age. The saving decision with reference to the post-productive age is one of the most important economic decisions of households, however, several studies found that there is a large portion of the population underestimating this decision. A hyperbolic consumer (especially a naïve one) tends to postpone this important decision. O'Donoghue and Rabin (1998) demonstrate that people tend to save their life-time savings using certain financial instruments even if they could reach a higher yield with no additional risk. However, consumers do not sacrifice immediate costs even if they know that they reduce their future yields and that the immediate costs are negligible compared to possible future utility gains. O'Donoghue and Rabin (1998, 2000) present the intuitive result that naïve hyperbolic consumers may theoretically delay strategic savings decision permanently. The notion of the hyperbolic intertemporal utility function may also partially explain huge popularity of the *obligatory* state social security system.[9] However Parker and Preston (2002) or Hurd and Rohwedder (2003) demonstrate that the consumption decline is not often caused only by insufficient savings. The decline in consumption

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[8] We think that just self-awareness and deeper understanding of our own problems with self-control and procrastination is not sufficient for transforming a naïve consumer into a sophisticated one. Even knowledge of the problem does not necessarily imply the ability to overcome it. Further, we disregard the fact that the problem of self-control greatly varies upon different actions (goods).

[9] Even on risk-adjusted basis

is often expected by consumers and is linked to a change in life-style – different structures of outlays, substitution of expenses by home production, different consumption of leisure.

Hyperbolic consumers (without access to a commitment technology) tend to ‘over-consumption’, i.e. higher future consumption than they considered as optimal in the past. This fact has important implications for life-cycle theory. For hyperbolic consumers the existence of a commitment technology is crucial. Therefore the government and financial markets are motivated to provide such instruments that can serve as a commitment. Sophisticated hyperbolic consumers are ready to pay for such services – term deposits with high penalties for early withdrawal, pension funds etc.

Laibson (1996, 1997a, 1997b) or Harris and Laibson (2001) treat the problem of a sophisticated consumer as a game among individual selves. A consumer’s *t-1* self is aware of a possible future preference reversal and seeks for a commitment. The commitment technology in the model is the accumulation of illiquid assets. These are viewed as an implicit, however imperfect, commitment technology because illiquid assets manipulation is subject to transaction costs and therefore cannot be easily splurged in future.

Laibson et al. (1998) and Angeletos et al. (2001) investigate the differences between the life cycle of exponential and sophisticated hyperbolic consumers. Numerical simulations[10] compare evolution of savings and illiquid assets holding. Consumers may accumulate liquid and illiquid assets and are allowed to use credit cards. Angeletos et al. (2001) show that the consumption profiles of exponential and (sophisticated) hyperbolic consumers don’t differ dramatically. However, hyperbolic consumers at the start of the consumption life use more borrowings from credit cards and at retirement age the consumption drops. The reason is the fact that hyperbolic consumers hold a large portion of their financial wealth in illiquid assets, which cannot be sold immediately without large certain costs and limit possibilities of consumption smoothing. A major difference between behaviour of the above-mentioned consumers is the proportion of financial wealth held in illiquid assets. For hyperbolic consumers these assets have additional feature of commitment and that is why they accept the costs of holding them.

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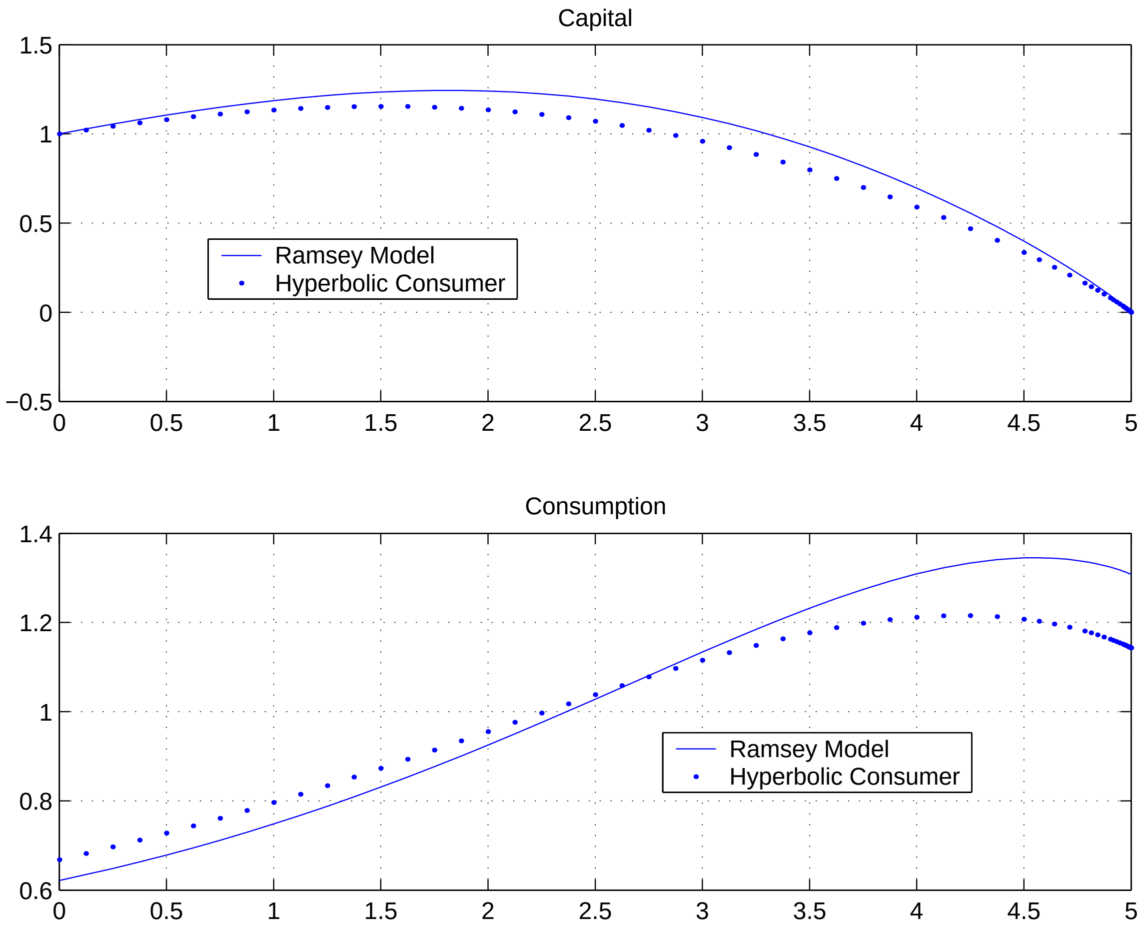
[10] The parameters of the simulations are very realistic. There is labor income uncertainty, consumption age 20—63 years, max length of life is 90 years. Credit card instruments disposable accompany liquid and illiquid assets for households.

These conclusions enrich life cycle hypothesis and bring it closer to observed stylized facts – households use credit cards, hold illiquid assets and join pension programs with high penalties for early withdrawals. Holding illiquid assets limits consumption smoothing even in the case of expected fluctuations of labor income, etc. It implies relatively close co-movement of consumption and disposable income.

In Fig 1 we present a simulation of standard Ramsey's optimal growth model with a naïve hyperbolic consumer compared to exponential one. The trajectory of consumption and savings is typical for these consumers, as the hyperbolic one is time inconsistent.

Normative analysis of hyperbolic consumer models is more problematic, there is no consensus on criteria of analysis (Harris and Laibson, 2001). The problem is, which 'self' should form the basis of the analysis if we consider utilitarian framework. Laibson (1997a, 1997b) applies Pareto efficiency criteria. If there is such a program that allows an improvement of utility for all selves, then this program is Pareto efficient. Sophisticated hyperbolic consumers will seek for commitment instruments. Even naïve consumers would appreciate being committed to a certain program, however not at the moment they were forced.

Fig 1: Ramsey Growth Model with naïve hyperbolic consumer



Source: own calculation

Hence it may be tempting for a government with certain economic policy objectives to arrange commitments of consumers, especially in the case of naïve hyperbolic consumers. Of course, that is very dangerous for many reasons. In contrast, the government could support these objectives by enhancing market instruments acting as a commitment technology. The government can of course set up the obligatory social security system with all its pros and cons. Imrohoroglu et al. (2000) simulate a model with sophisticated agents in which social security can serve as a substitute of other mechanisms of commitment. However the existence of the obligatory system distorts financial markets and labor supply decision of households.

V. Environmental economics

Environmental economics is primarily concerned with long-run issues and thus it is not surprising that the discounting is an important issue. Indeed, the most of environmental discussion is cast in terms of the damaged planet we would leave to our offspring. Therefore for assessment of optimal level of pollution, natural resource exhaustion etc. it is necessary to have a methodology for comparing the utilities of different generations.

As Aghion and Howitt (1999) argue the requirement that we should leave the environment to our offspring in the same state as we have obtained it makes little sense. There is an issue of substitution of different kinds of productive inputs and the environment (or the natural resources) are only a part of them. Developed countries have succeeded in diminishing child mortality, eliminating illiteracy, increasing leisure and the expected life-time not because we have a better environment (which is at least questionable) or we have more exhaustible resources (which is certainly not true) than preceding generations, but because we know how to use available resources more effectively. Human capital can substitute physical capital or natural resources. However there are certain actions of the current generation, which can impose huge costs on future generations. A spectacular example is nuclear waste: because of nuclear energy a current generation can enjoy cheap electricity and need not rely on carbon plans, which have serious negative impacts on the environment, but future generations can suffer from the non-negligible costs of nuclear waste treatment.

Thus there is a need for a normative criterion for the time valuation of utility. In other words there is a need of a robust and widely accepted form of discounting. Mainstream economists assume the exponential discounting function (2a) with $0 < \beta \leq 1$ as such a criterion. There is a bulk of models solving for an optimal environmental policy under such an assumption. For example the pioneering contribution of Hotteling (1931) in resource exhaustion is an example of such approach. Hotteling's analysis suggests that the rate of exhaustion should be equal to the parameter β. This is intuitive: it is well known that a Pareto efficient allocation under the exponential discounting satisfies[11]:

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[11] Under certain assumptions (e.g. interior solution).

$$mu(c_t) = \beta^{-1} R_t mu(c_{t+1}), \tag{14}$$

where *mu* is marginal utility and R_t is a cost of shifting a unit of resources between times t and t+1. $R_t < 1$ implies that it is costly to shift resources between consecutive periods, while $R_t > 1$ means that resources can be accumulated[12]. Formula (14) is an intertemporal analogy of the Gossen law, which dictates that the ratio of marginal utility to price is constant among the goods in the optimum.

For illustrative purposes, consider the following simple model of non-renewable resource exhaustion: Robinson Crusoe on an isolated island has x units of a non-renewable resource and his life horizon is T ($T = \infty$ is allowed also). His intertemporal valuation is given by the exponential discounted sum (2a) with a parameter $0 < \beta < 1$ and the instantaneous utility function has the standard CRRA form (for $\sigma = 1$ this utility function is defined to be logarithmic) i.e.:

$$U({}_t x) = \sum_{t=1}^{T} \beta^{t-1} \frac{x_t^{1-\sigma} - 1}{1-\sigma}.$$

The resource constraint implies that $x \geq \sum_{i=1}^{T} x_i$ and the non-negative consumption condition dictates that $x_i \geq 0$. The solution to this problem is simple and it can be easily shown that it satisfies $\beta x_t^{-\sigma} = x_{t+1}^{-\sigma}$ (compare to 14) and from the resource constraint we have (for $\overline{\beta} = \beta^{\frac{1}{\sigma}}$):

$x_t = \overline{\beta}^{t-1} \dfrac{(1-\overline{\beta})x}{(1-\overline{\beta}^{T+1})}$ if his horizon is finite,

$x_t = \overline{\beta}^{t-1} (1-\overline{\beta})x$ when $T = \infty$.

However as argued in Section 3, exponential discounting is not necessarily an appropriate approach to consumer behavior modeling for empirical reasons.

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[12] A possible interpretation of the variable R_t in formula (14) may be the following one: the case $R_t < 1$ models situations when the resource decays or when the exhaustion costs are decreasing the total stock of the resource (the less of the resource there is, the more costly it is to extract it). On the other hand the case $R_t > 1$ models situation with a renewable resource.

Thus consider Robinson Crusoe endowed with quasi-hyperbolic preferences as (12). His ex-ante preferences would be:

$$U({}_t x) = \frac{x_1^{1-\sigma} - 1}{1-\sigma} + \delta \sum_{t=2}^{T} \beta^{t-1} \frac{x_t^{1-\sigma} - 1}{1-\sigma} \text{ pro } 0 < \delta \leq 1 .$$

It is easy but cumbersome to find the ex-ante optimal consumption pattern for such preferences:

$$x_t = \begin{cases} x - x_d & \text{for } t = 1 \\ \bar{\beta}^{t-2} \dfrac{1-\bar{\beta}}{1-\bar{\beta}^{T-1}} x_d & \text{for } t \geq 2 \end{cases},$$

when we use the definition $x_d = \bar{\beta}\bar{\delta} \dfrac{1-\bar{\beta}^{T-1}}{\left\{ (1-\bar{\beta})^{\sigma} + \bar{\beta}\bar{\delta}[1-\bar{\beta}^{T}]^{\sigma} \right\}^{\frac{1}{\sigma}}}$. Figure 2 displays ex-ante optimal consumption path for „exponential“ Robinson and two „hyperbolic“ Robinsons. The parameters used are $x = 1$, $\sigma = 1$, $\beta = 0.99$ a T = 10 and $\delta = 1$ for „exponential“ Robinson and $\delta = 0.9$, 0.7 res. for „hyperbolic“ Robinsons.

Environmentalists however challenge exponential discounting especially from the normative point of view. A representative voice of this kind is that by Chichilinski (1996) who argues that exponential discounting represents a dictate of the current generation over future ones, especially over those who live in the far-future. The logic behind this kind of argumentation is that even a huge cost or benefit would be negligible if discounted by a long time horizon. Thus it is a good thing to transfer huge costs on far-future generations according to criteria based on the exponential discounting, which does not sound to be ethically defensible. On the other hand, the solution is not to abandon the discounting for policy evaluation at all: this would leave the current generation with little resources. This problem was attempted by Li and Lofgren (2000) who propose a compromise, which consists of a convex combination of the exponential discounting and not discounting at all. However, note that this would not lead to an exponential discount factor, which would be a convex combination of the original one and zero, instead the new discount factor become time-dependent. Indeed:

$$-\ln\left\{\lambda e^{-\rho t}+(1-\lambda)e^{-\sigma t}\right\}\neq-(\lambda\rho+(1-\lambda)\sigma)t\,, \qquad (15)$$

where λ is a convex weight, ρ and σ are discount factor with $\sigma \rightarrow 0$ in the Li and Lofgren (2000) proposal. Note that the resulting discount factor (see left hand side of 15) is time-dependent and for $\sigma = 0$ it replicates the features of hyperbolic discounting.

Fig 2: *Robinson Crusoe Economy – finite horizon*

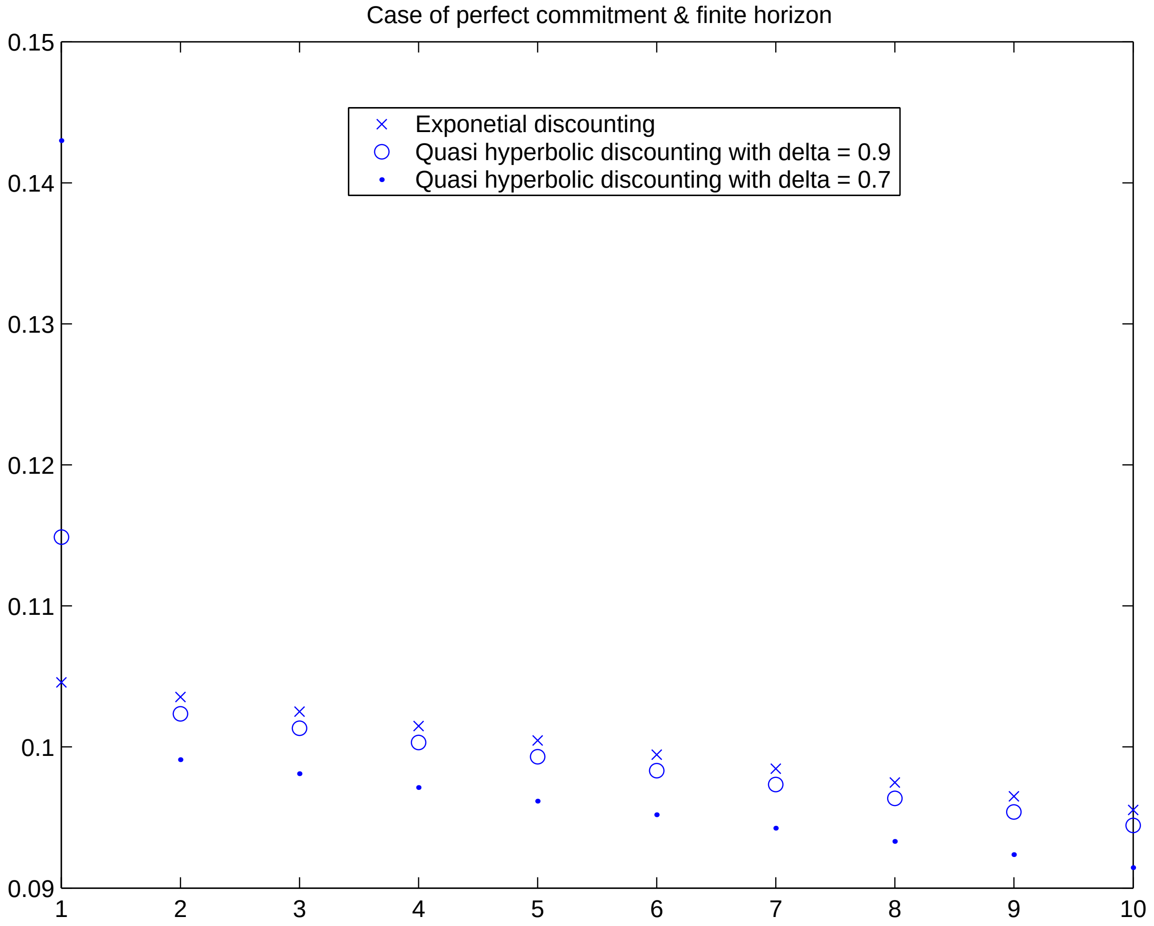
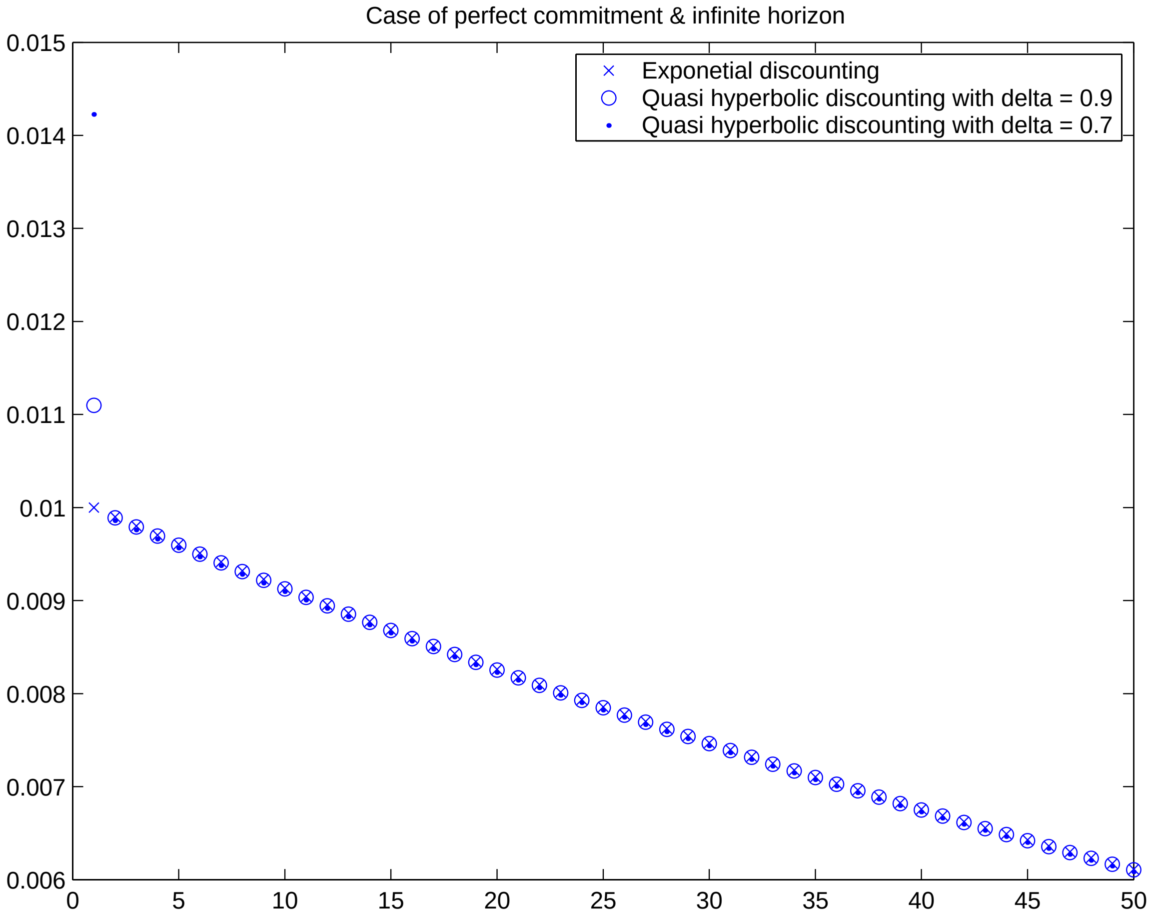


Fig. 3: *Robinson Crusoe Economy – infinite horizon*



Another normative criticism of the exponential discounting is due to Weitzman (1998). He argues that the fundamental uncertainty of future states of the world disable to determine the 'right' discount factor. He demonstrates that the discount rate should fall as a response to uncertainty. The intuition behind this can be illustrated using a simple example (analogy to discounting cash flow): consider an investment of one unit of a currency for T years. Let there be two states of the world: interest rate can be either 1% p.a. or 7% p.a. with equal probability. If an average interest rate 4% p.a. is used to compute the expected investment return, the return would be 1.04^T, which is for $T = 10$ res. 100 res. 200 years 1.48 res. 50.50 res. 2550.70 units of the currency. However this approach is completely wrong. The correct approach to computation of the expected return is $0.5\times1.01^T + 0.5\times1.07^T$, which for $T = 10, 100, 200$ let yields 1.53, res. 435.20 res. 3764.70 units of currency. Thus for long-horizons, scenarios with the higher interest rate have a larger impact.[13] Similar ideas apply to utility discounting: if there is an uncertainty about the ‘right’ discount rate, the relative importance of scenarios with low discount rates rise. Since it is natural to assume that more far future involves more uncertainty than close future, far future should be discounted using smaller discount factor than close future; thus Weitzman (1998) replicates the normative criterion by Li and Lofgren (2000). Provided that discount rates can be described as a random walk type of stochastic processes, then it holds that asymptotically far future should be discounted with the lowest possible rate. Newell and Pizer (2003) apply Weizman’s ideas on the term structure of U.S. interest rates (under a salient assumption that interest rates reflect the discount factor). They find that a valuation of the greenhouse gases climate change would change by a factor of about two if the uncertainty of discounting would be dealt with.

Note that the non-exponential (hyperbolic) discounting was derived based on normative assessment. This is a different approach than used in the preceding sections, where the exponential discounting was criticized based on positive arguments (empirical regularities in observed consumer behavior).

VI. Conclusion

This paper discusses different approaches to intertemporal valuation, including the mainstream exponential discounting. However, the empirical evidence against this mainstream approach suggests that there is a launch for an alternative. Such an alternative may be represented by hyperbolic discounting. Also normative issues, especially those related to environmental economics, were dealt with and we show that also from the normative point of view the hyperbolic alternative can be an interesting approach.

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[13] From a technical point of view, this is a consequence of Jensen inequality, which implies that for a strict concave function f, non-degenerate random variable x and the expectation operator E the following inequality holds $\mathrm{E}f(x) < f(\mathrm{E}x)$, whenever integrals on both sides absolutely converge.

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